Artificial Intelligence

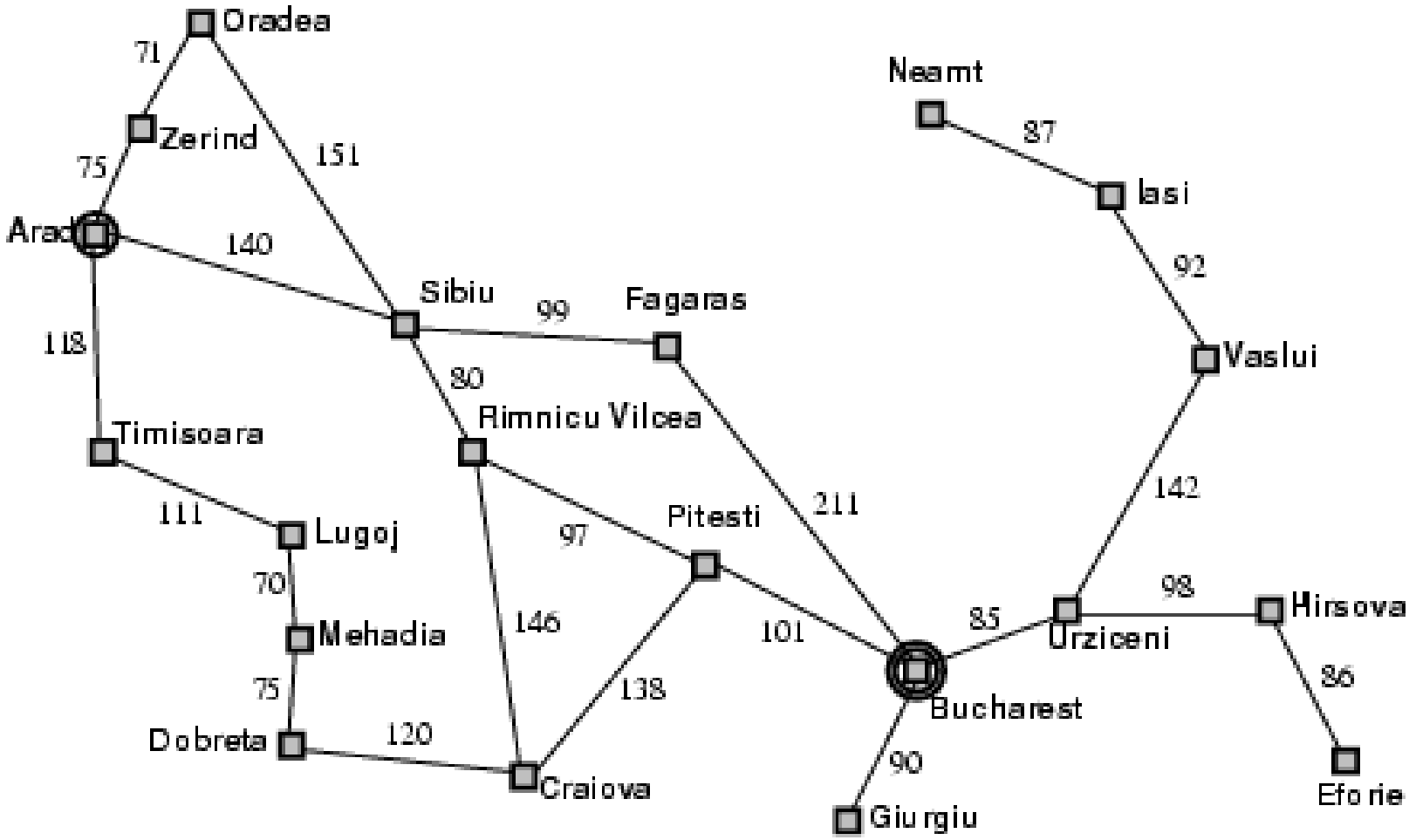
Introduction to Search

(Ch. 3.1-4)

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

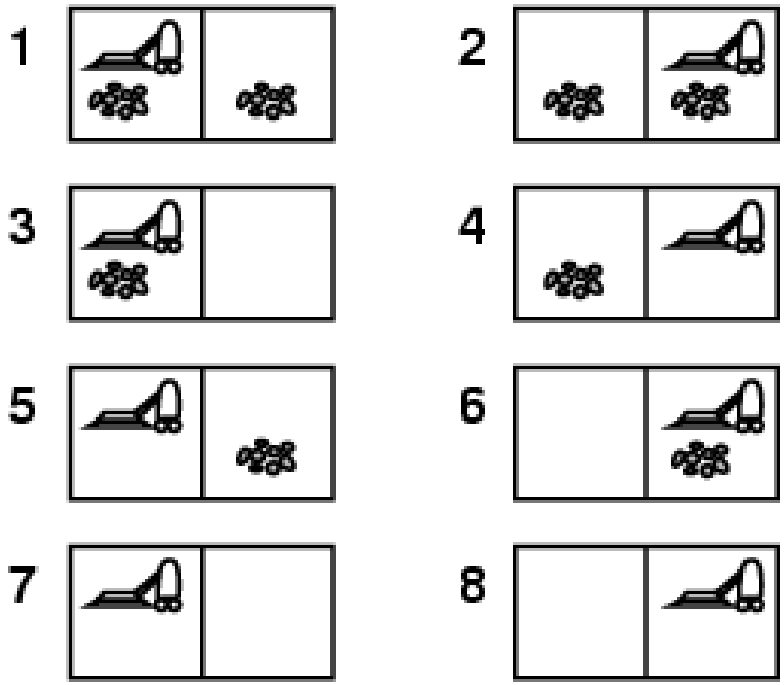


Problem types

- **Deterministic, fully observable → single-state problem**
  - Agent knows exactly which state it will be in; solution is a sequence
- **Non-observable → sensorless problem (conformant problem)**
  - Agent may have no idea where it is; solution is a sequence
- **Nondeterministic and/or partially observable → contingency problem**
  - percepts provide new information about current state
  - solution is a contingent plan or a policy
  - often interleave search, execution
- **Unknown state and action space → exploration problem**

Example: vacuum world

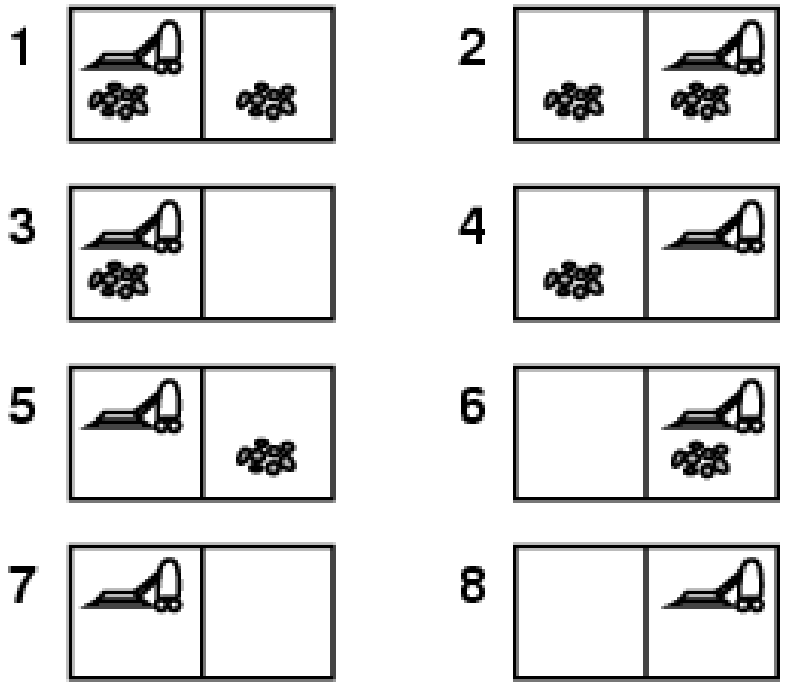
- **Single-state, start in #5. Solution?**
- **Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution?**



Example: vacuum world

- **Contingency**
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7

Solution?



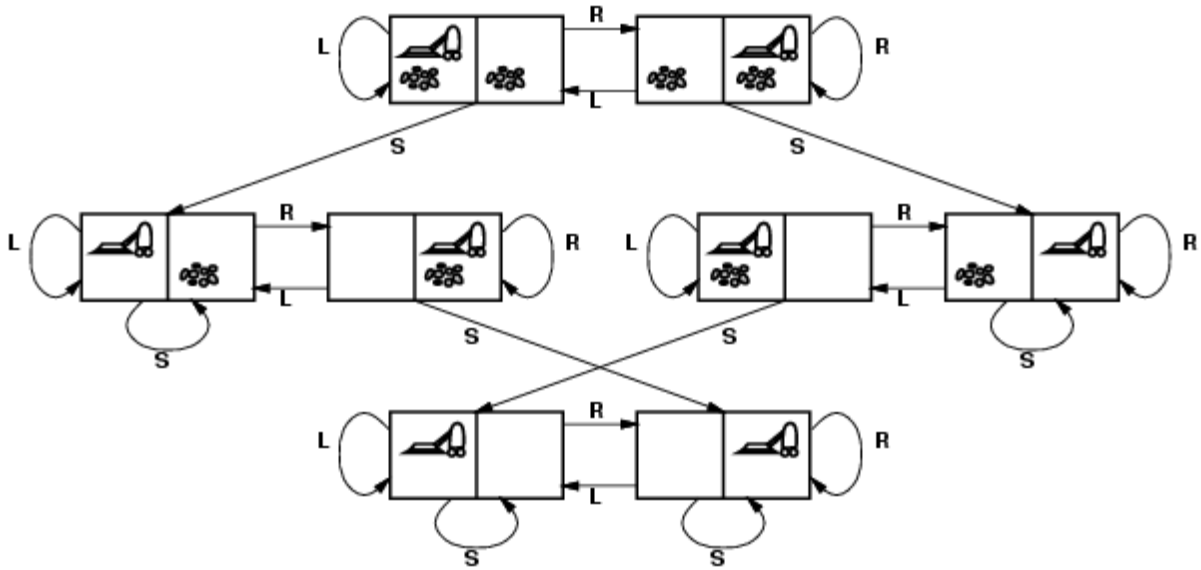
Single-state problem formulation

- A problem is defined by four items:
  1. initial state e.g., "at Arad“b
  2. actions or successor function *S(x)* = set of action–state pairs
     - » e.g., *S(Arad)* = {<*Arad* → *Zerind, Zerind*>, … }
  3. goal test, can be
     - » explicit, e.g., *x* = "at Bucharest"
     - » implicit, e.g., *Checkmate(x)*
  4. path cost (additive)
     - » e.g., sum of distances, number of actions executed, etc.
     - » *c(x,a,y)* is the step cost, assumed to be ≥ 0
- A solution is a sequence of actions leading from the initial state to a goal state

Selecting a state space

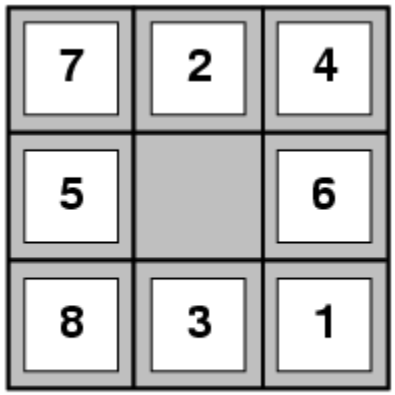
- (Abstract) Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
- (Abstract) solution = set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Vacuum world state space graph

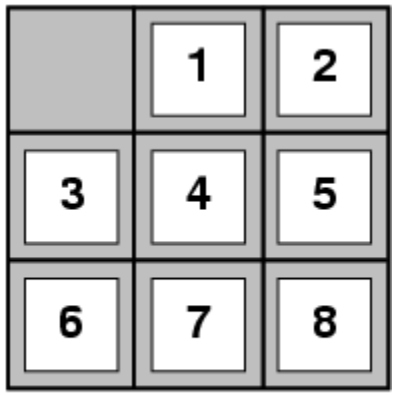


- states?
- actions?
- goal test?
- path cost?

Example: The 8-puzzle



Start State

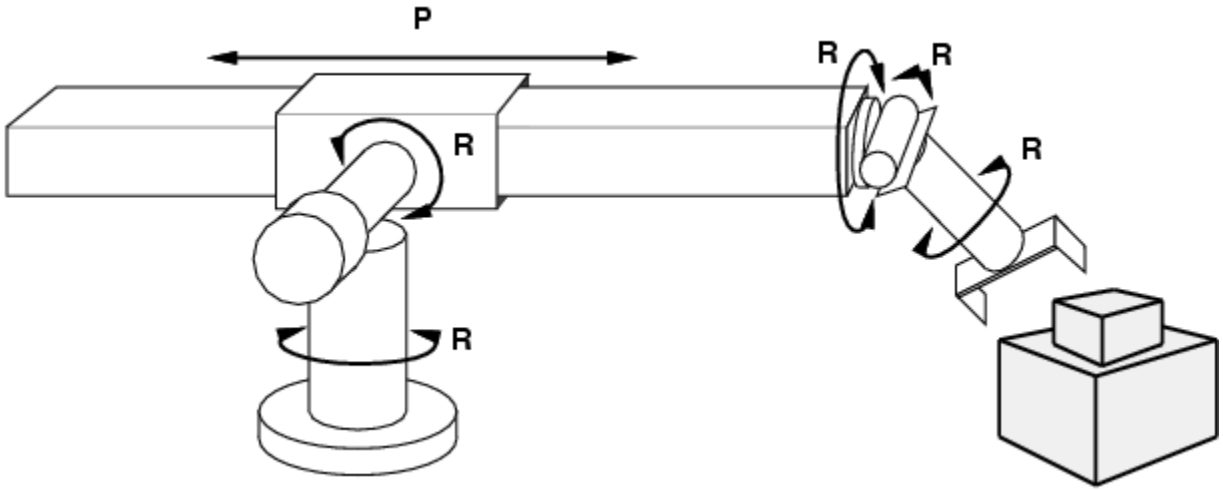


Goal State

- states?
- actions?
- goal test?
- path cost?

[Note: optimal solution of *n*-Puzzle family is NP-hard]

Example: robotic assembly



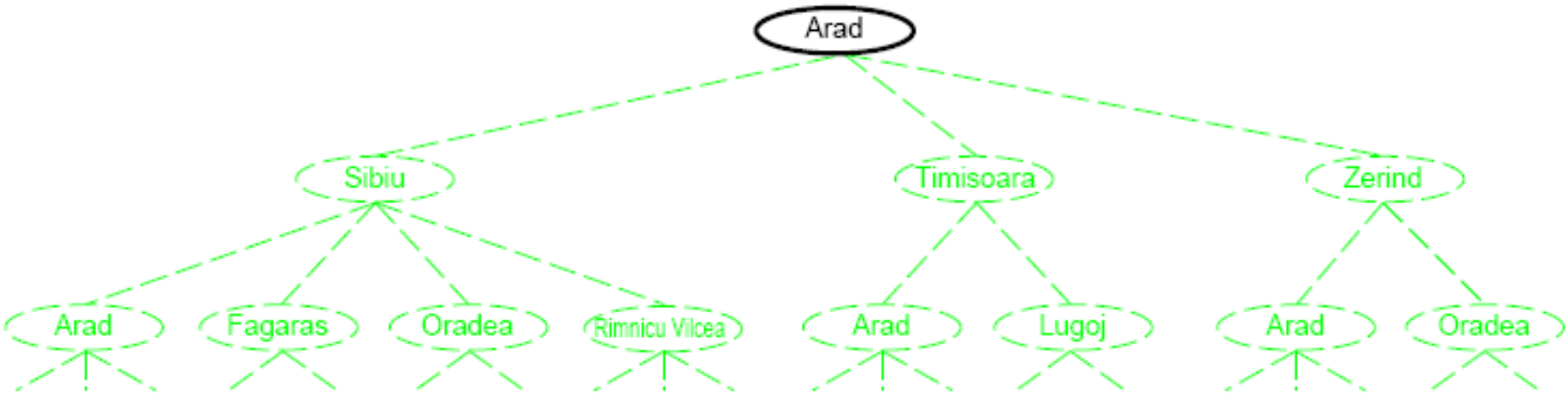
- states?:
- actions?:
- goal test?:
- path cost?:

Tree search algorithms

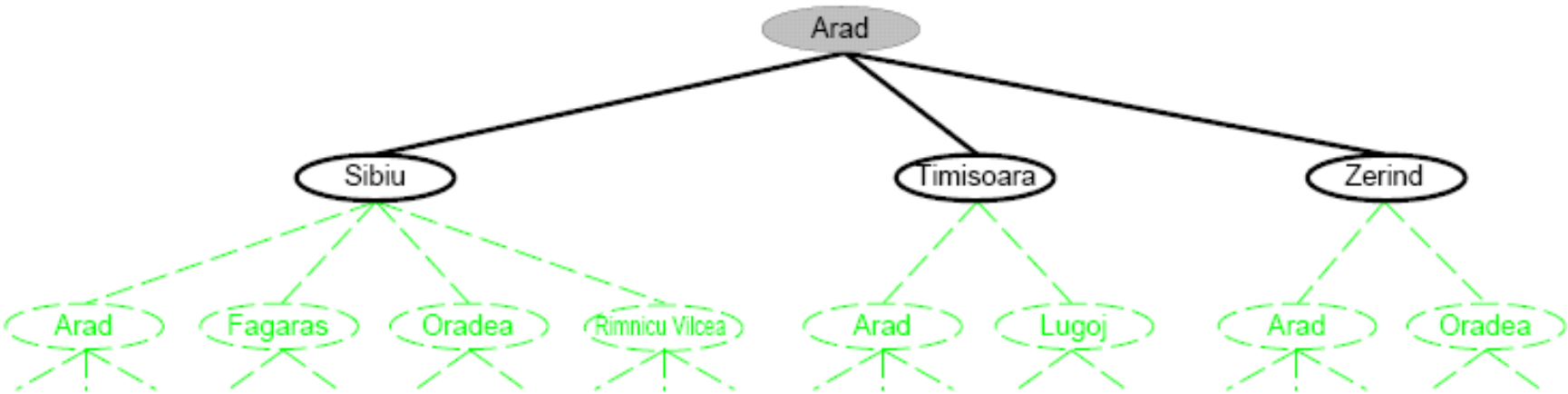
- **Basic idea:**
  - **offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. expanding states)**

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

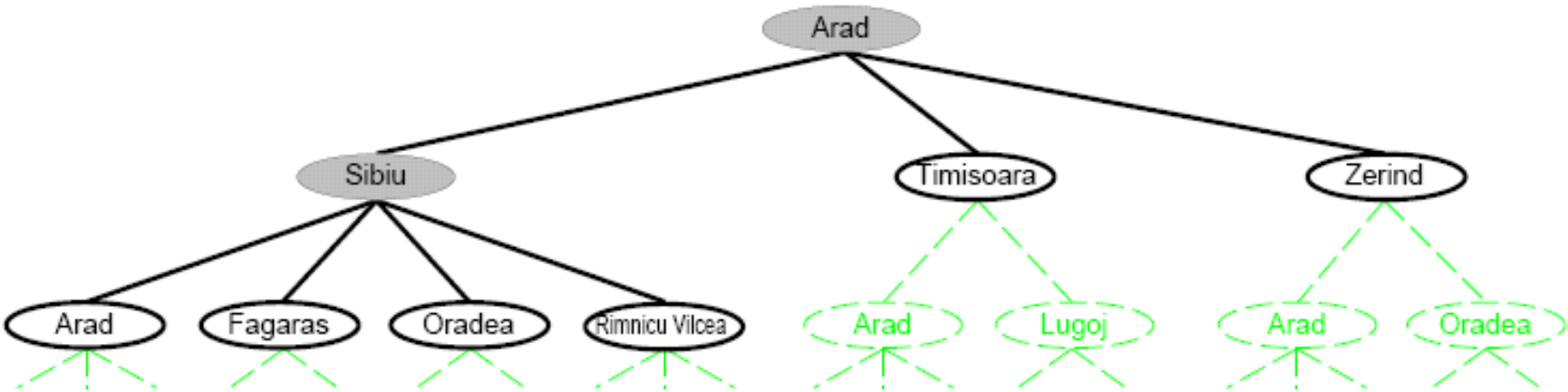
Tree search example



Tree search example



Tree search example



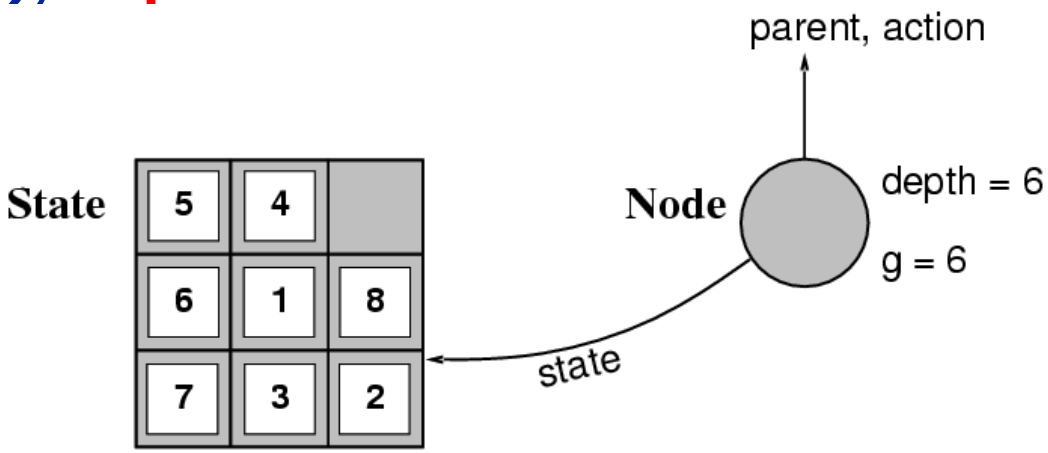
Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)

function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;  ACTION[s] ← action;  STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The Expand function creates new nodes, filling in the various fields and using the Successor-Fn of the problem to create the corresponding states.

Search strategies

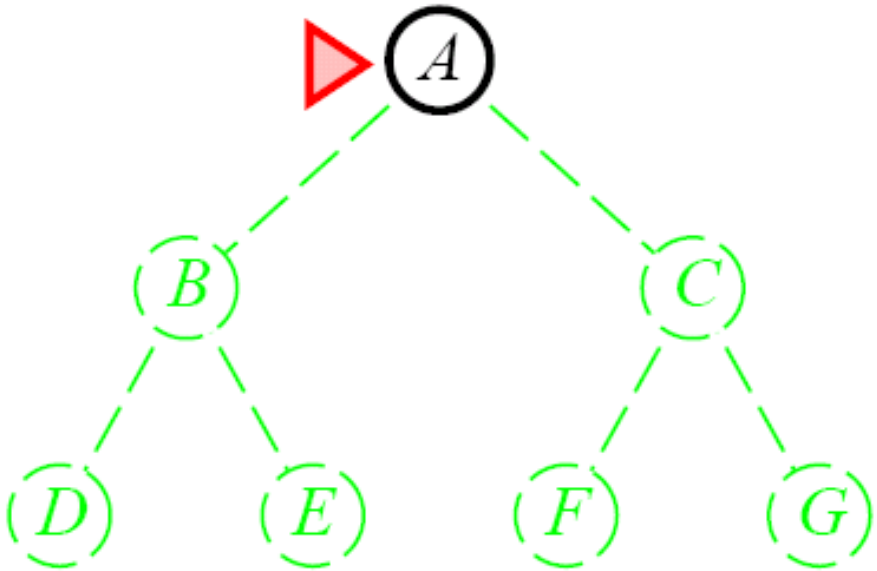
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - optimality: does it always find a least-cost solution?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Uninformed search strategies

- **Uninformed search strategies use only the information available in the problem definition**
  - **Breadth-first search**
  - **Uniform-cost search**
  - **Depth-first search**
  - **Depth-limited search**
  - **Iterative deepening search**

Breadth-first search

- **Expand shallowest unexpanded node**
- **Implementation:**
  - ***fringe* is a FIFO queue, i.e., new successors go at end**



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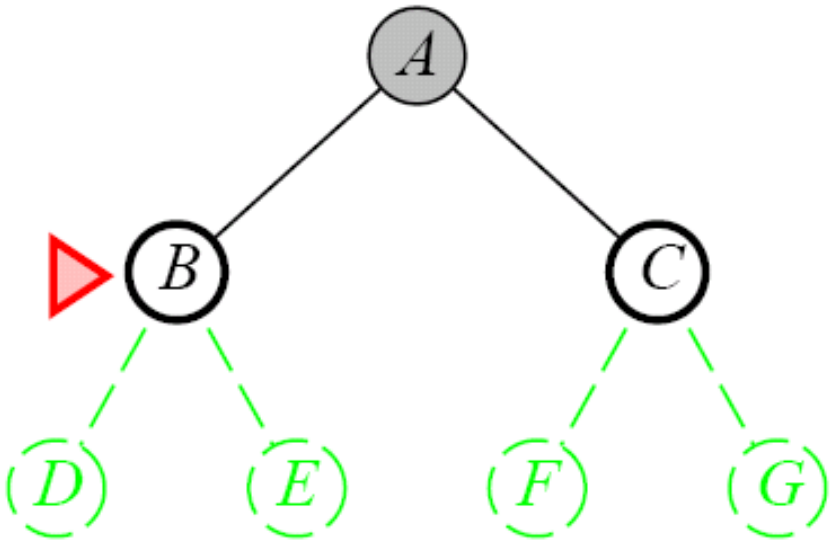

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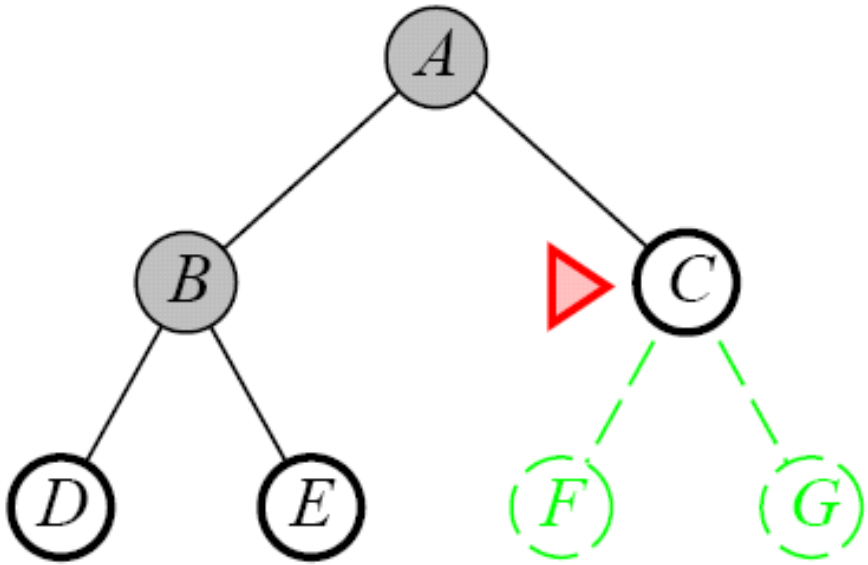

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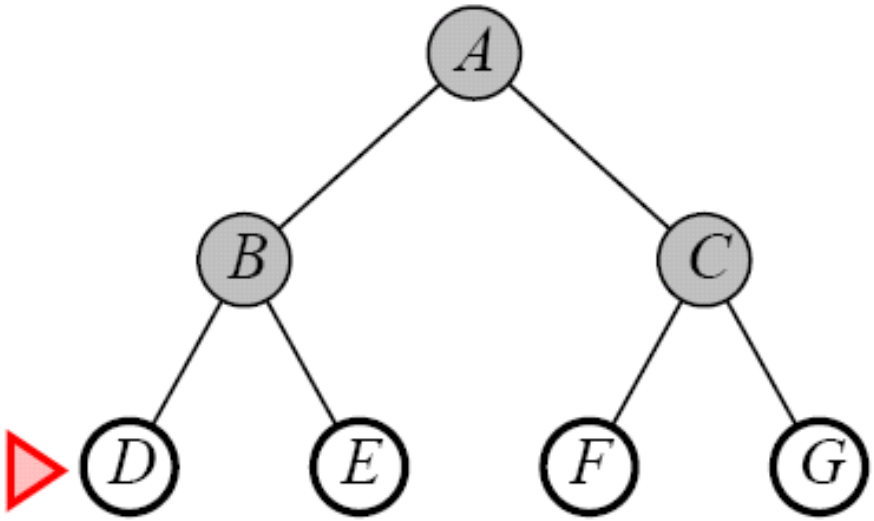


Properties of breadth-first search

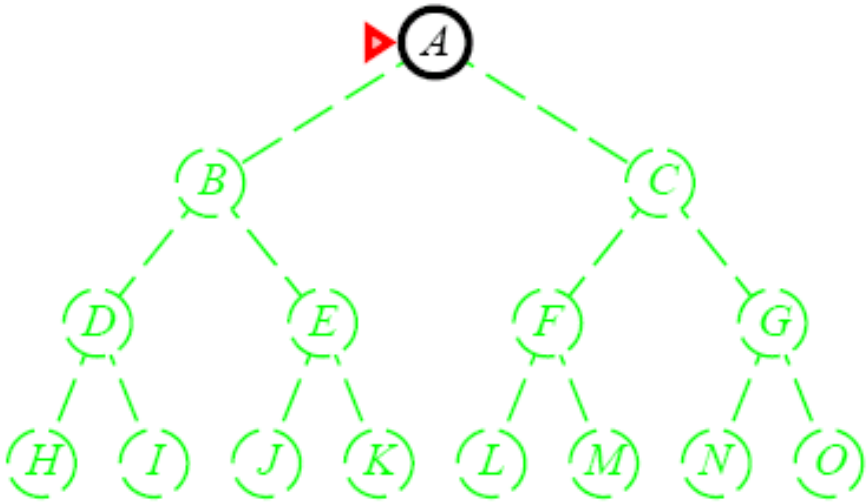
- Complete?
- Optimal?
- Time?
- Space?

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete?
- Optimal?
- Time?
- Space?

Depth-first search

- **Expand deepest unexpanded node**
- **Implementation:**
  - ***fringe* = LIFO queue, i.e., put successors at front**



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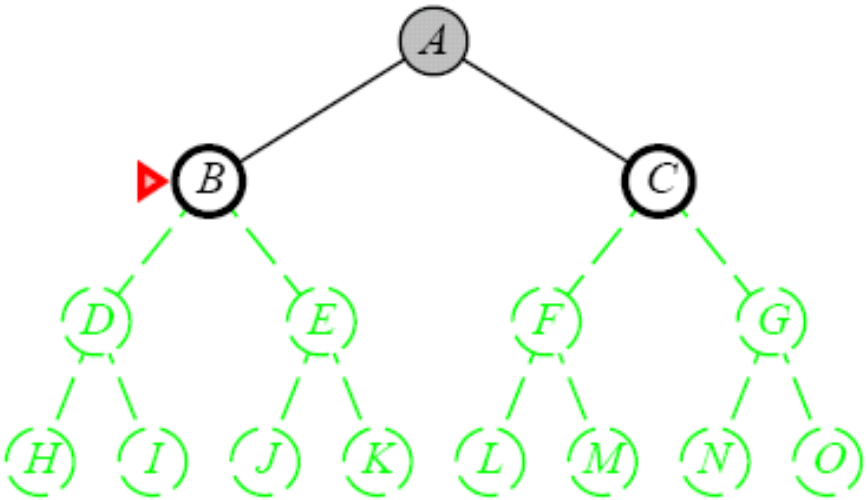

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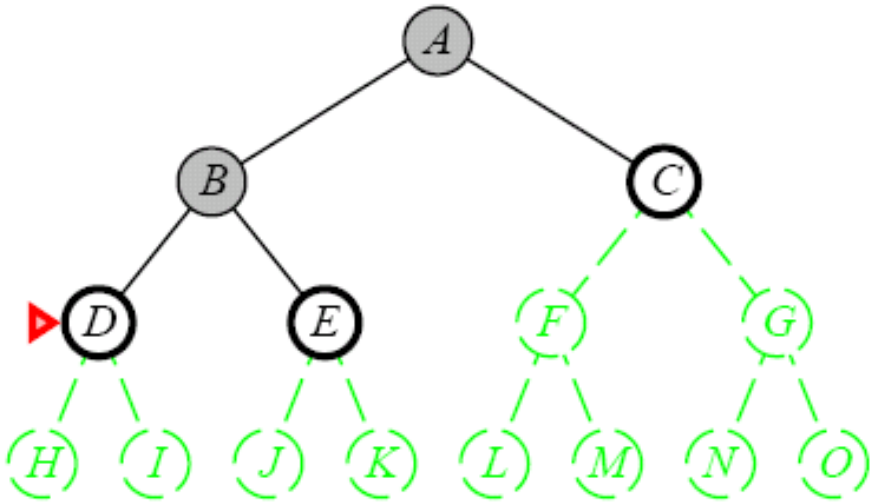

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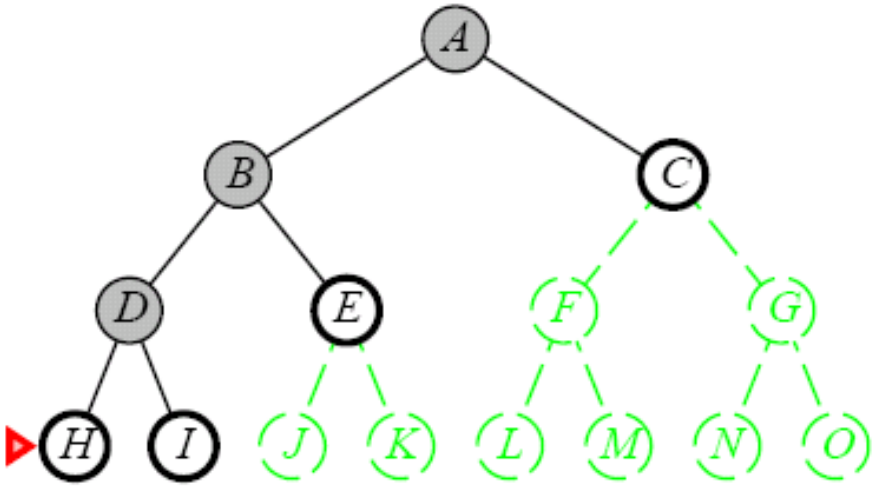

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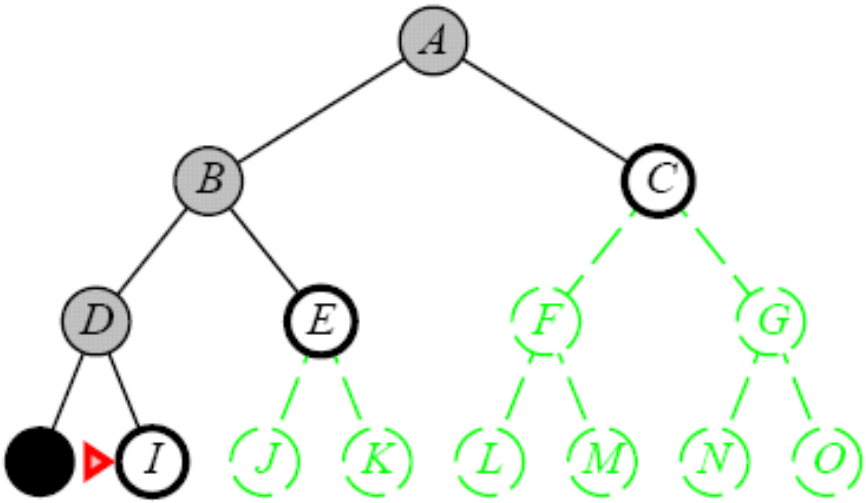

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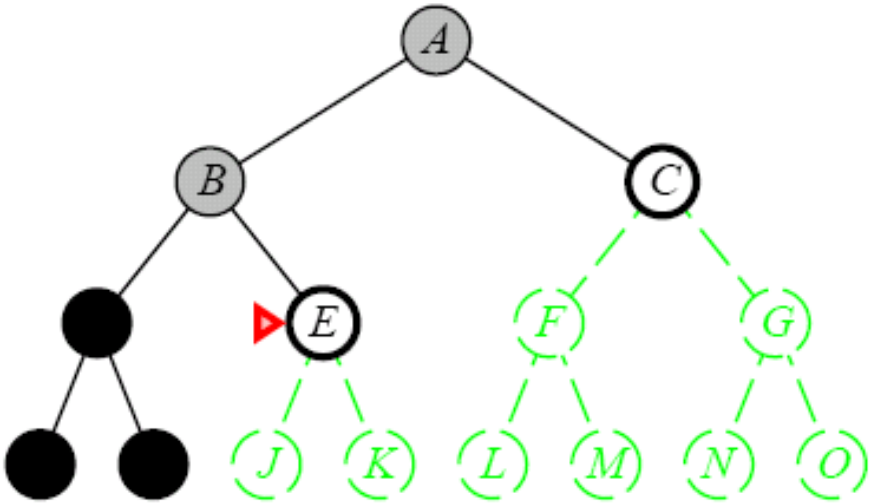

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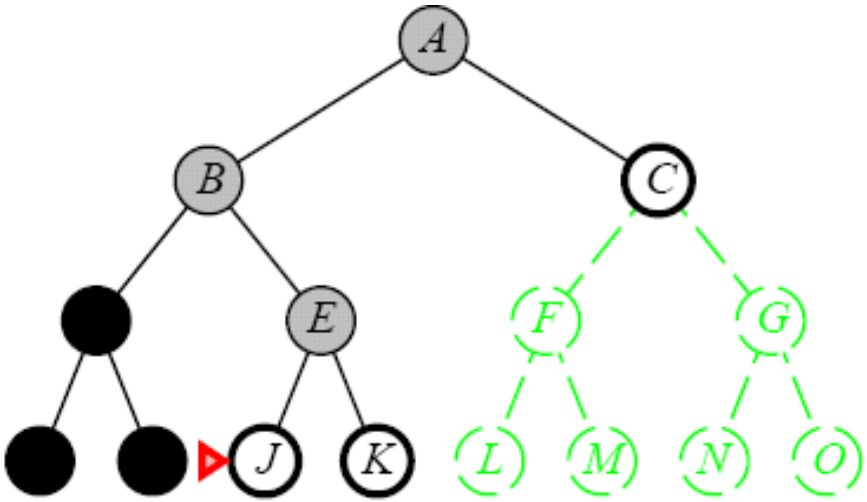

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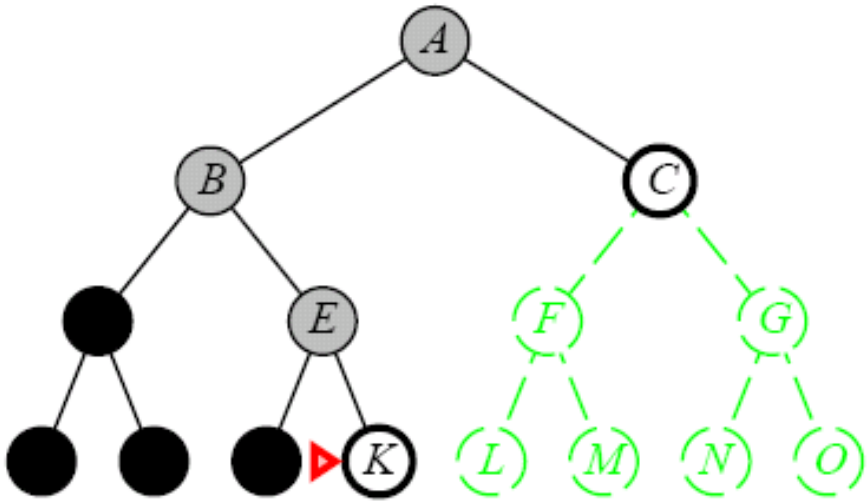

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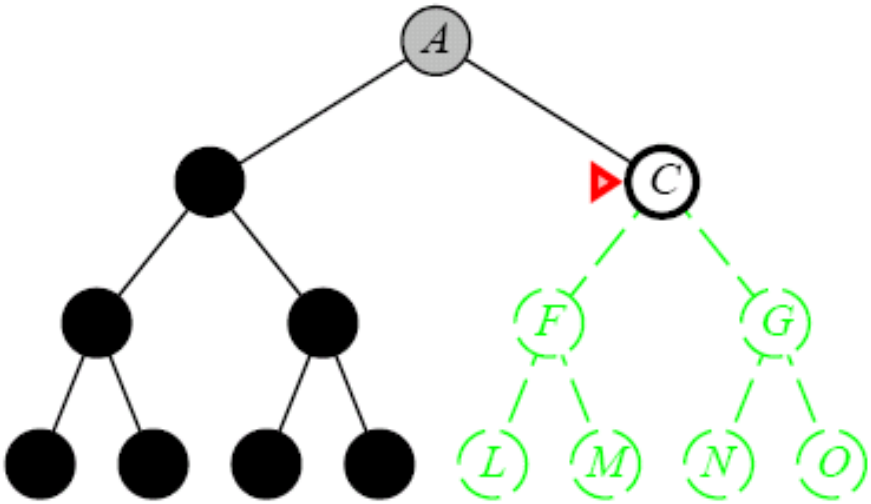

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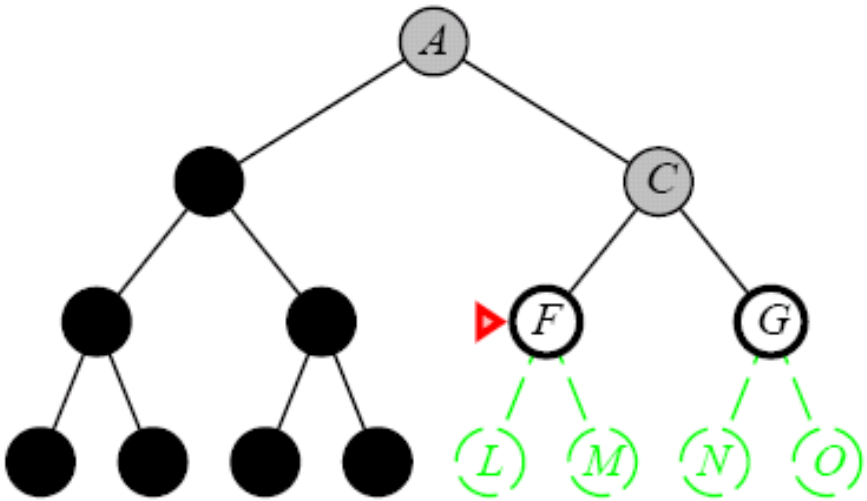

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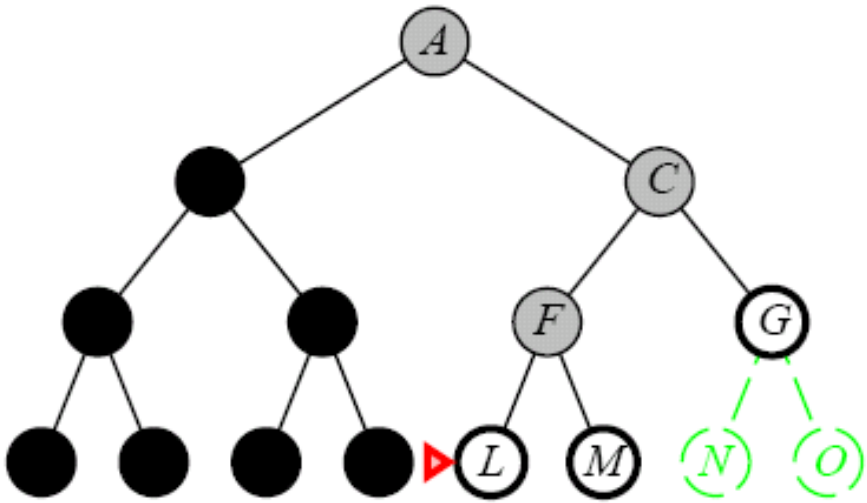

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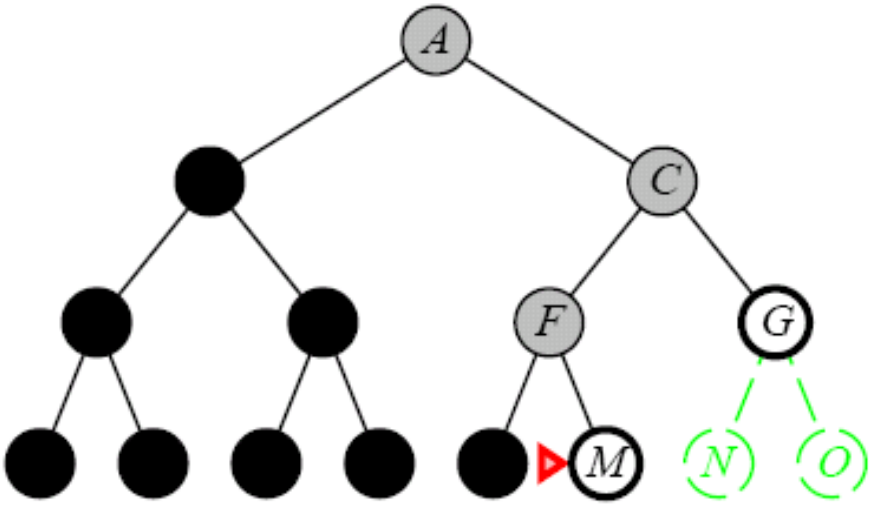


Properties of depth-first search

- Complete?
- Optimal?
- Time?
- Space?

Depth-limited search

- **Depth-first search with depth limit *l*, i.e., nodes at depth *l* have no successors**

- **Recursive implementation:**

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or failure
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

Iterative deepening search *l* =0

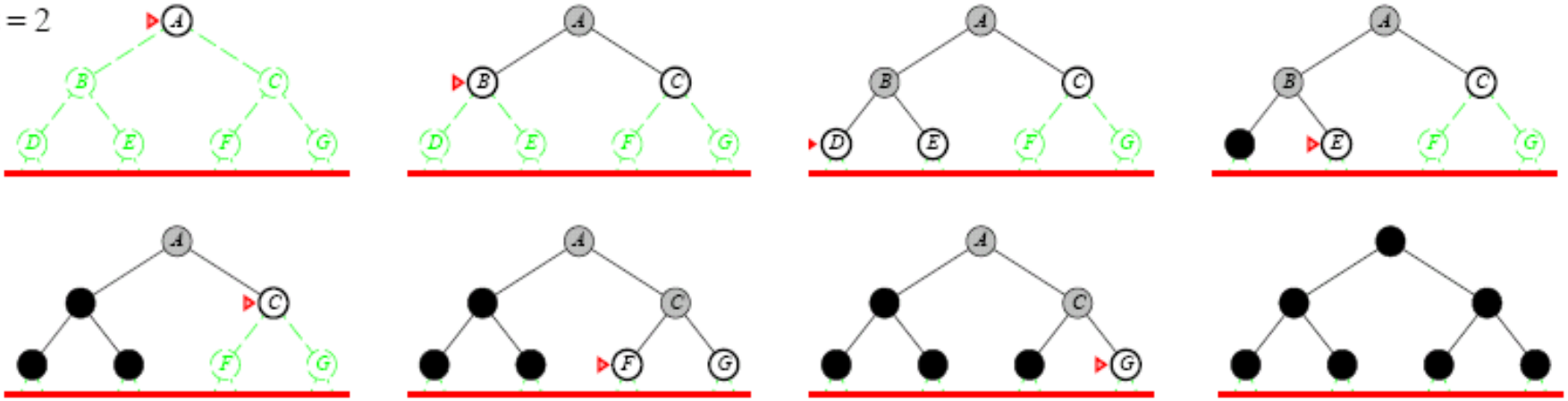
Limit = 0

Iterative deepening search *l* =1

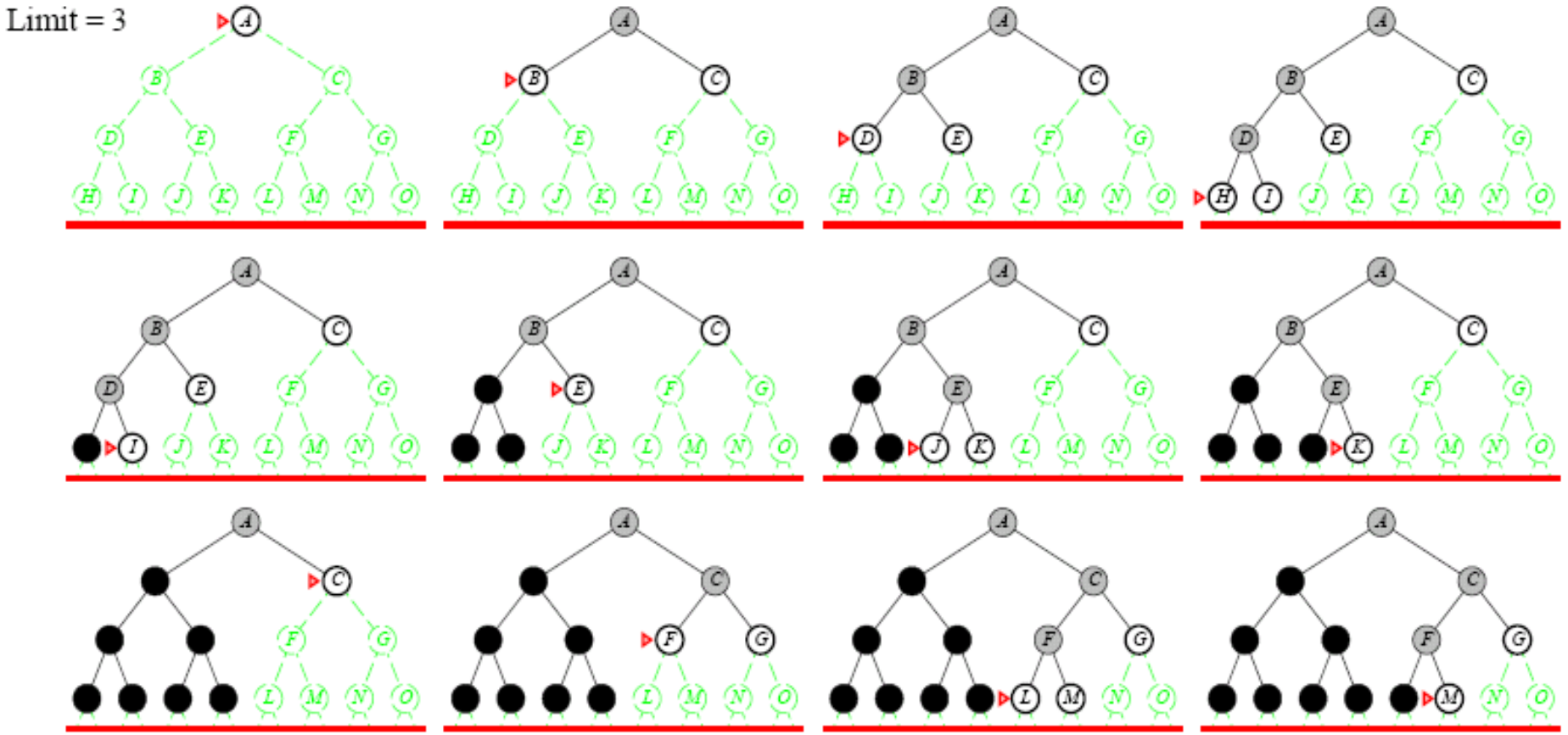
Limit = 1

Iterative deepening search *l* =2

Limit = 2



Iterative deepening search *l* =3



Iterative deepening search

- **Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:**

  N_{DLS} =

- **Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:**

  N_{IDS} =

- **For *b* = *10*, *d* = *5*,**
  - N_{DLS} =
  - N_{IDS} =

- **Overhead =**

Properties of iterative deepening search

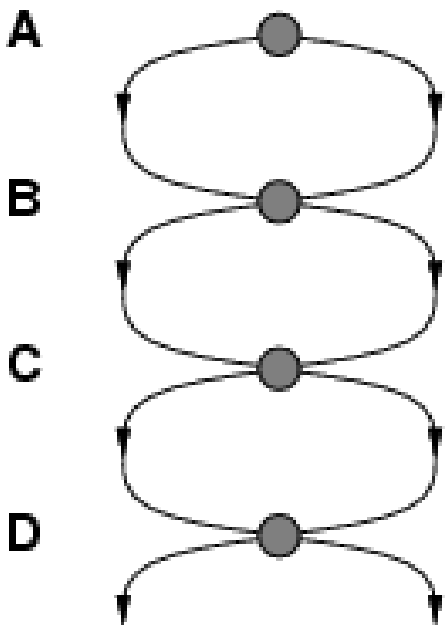
- Complete?
- Optimal?
- Time?
- Space?

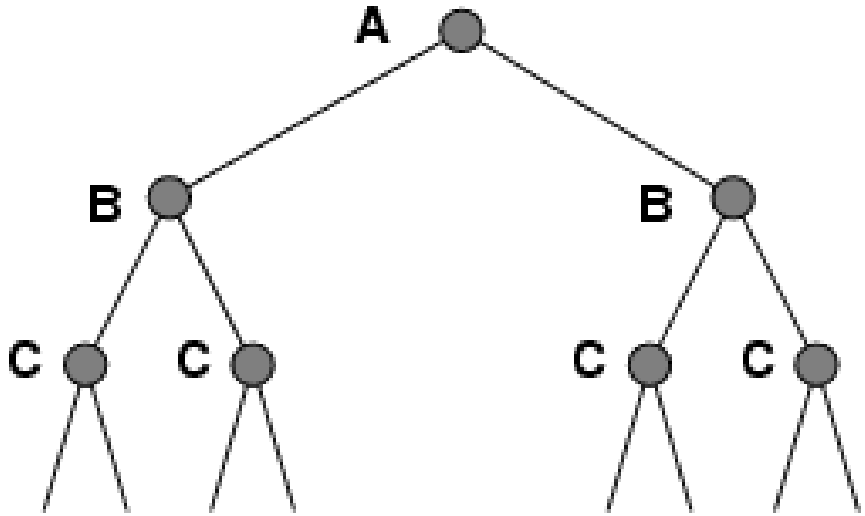
Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Repeated states

- **Failure to detect repeated states can turn a linear problem into an exponential one!**





Graph search

- **The only difference is detecting repeated states**

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms
- Graph search can be exponentially more efficient than tree search