Artificial Intelligence

Heuristic Search

(Ch. 3.5-6)

Best-first search

- **Idea: use an evaluation function *f(n)* for each node**
  - **estimate of "promisingness"**

→Expand the most promising unexpanded node

- **Implementation:**

  Order the nodes in fringe in decreasing order of promisingness

- **Special cases:**
  - **greedy best-first search**
  - **A* search**

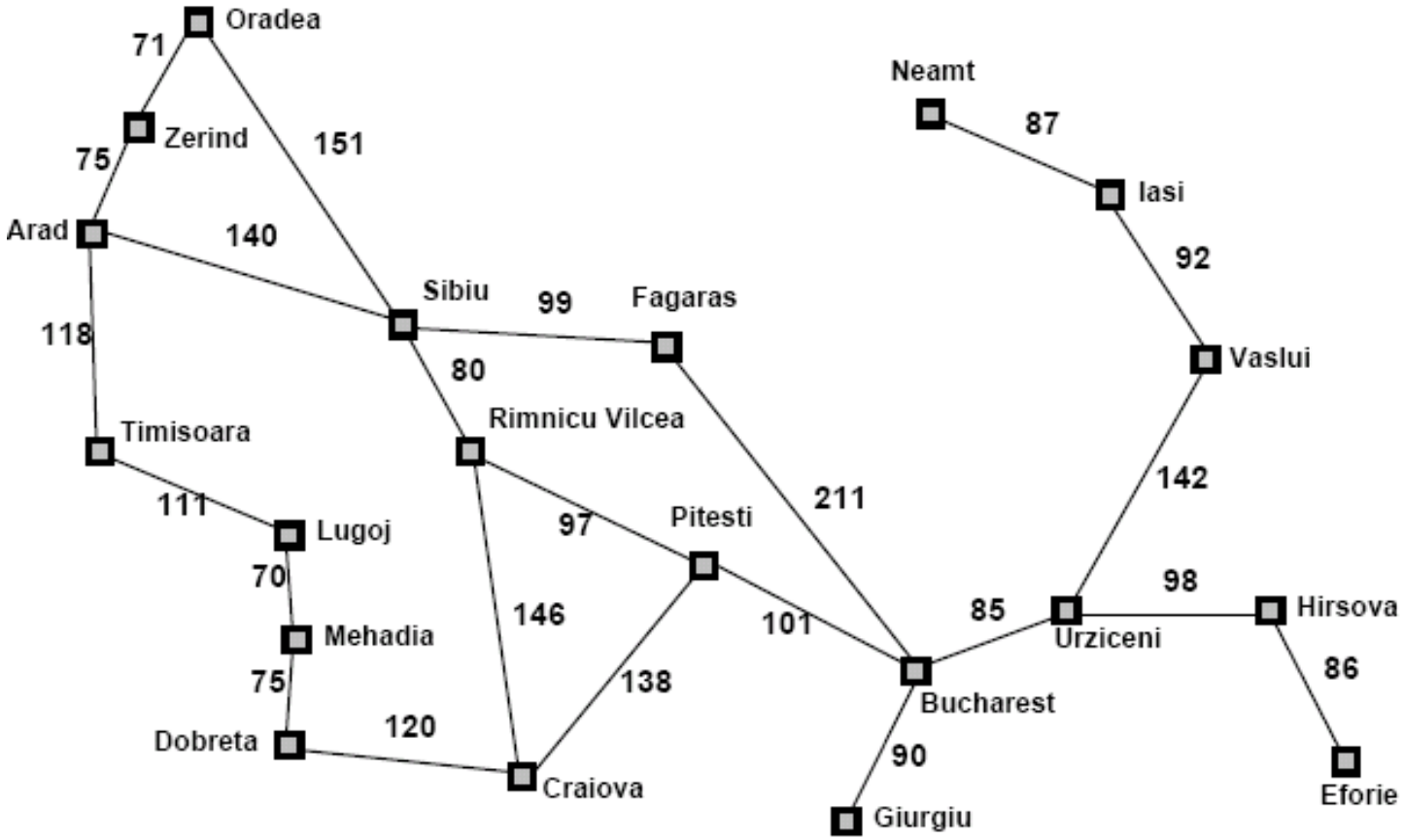
Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
  = estimate of cost from n to *goal*

- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

- Greedy best-first search expands the node that appears to be closest to goal

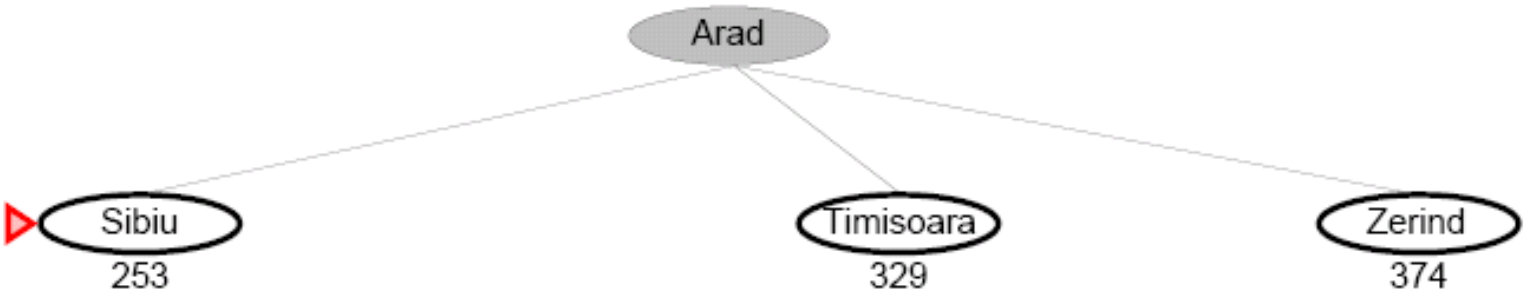
Romania with step costs in km



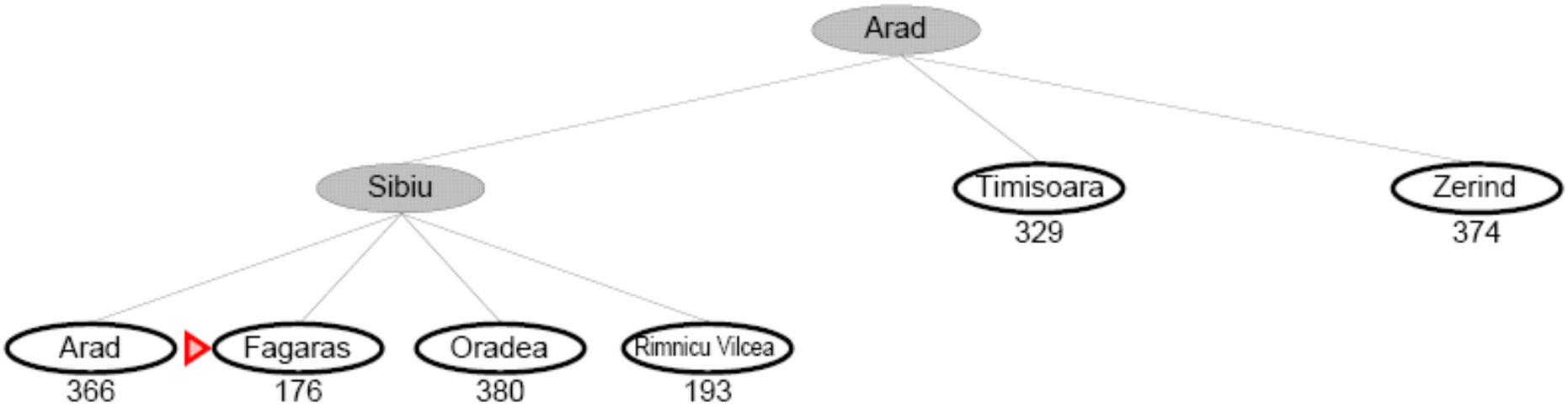
Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search example

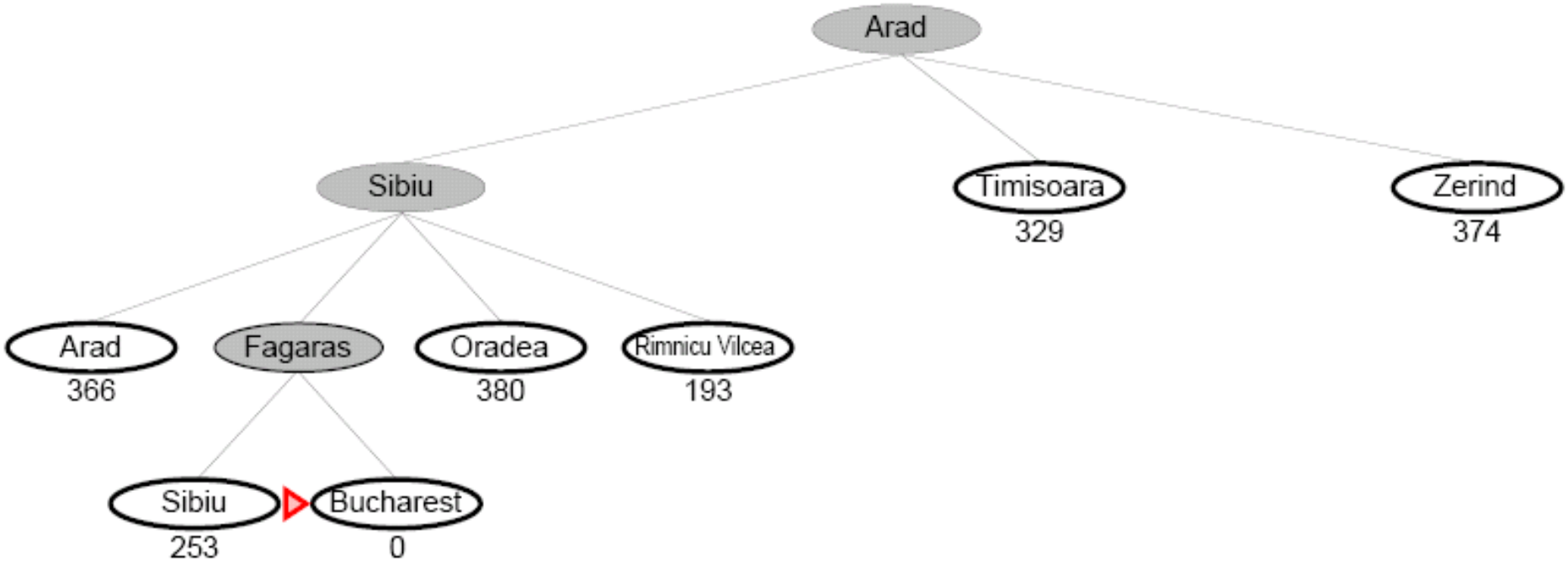
Greedy best-first search example



Greedy best-first search example



Greedy best-first search example



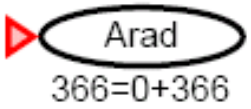
Properties of greedy best-first search

- Complete?
- Optimal?
- Time?
- Space?

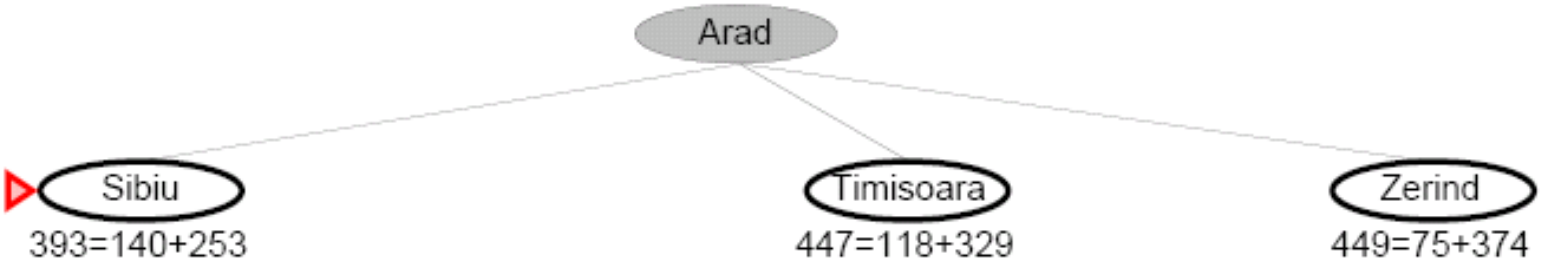
A* search

- **Idea: avoid expanding paths that are already expensive**
- **Evaluation function *f(n)* = *g(n)* + *h(n)***
  - ***g(n)* = cost so far to reach *n***
  - ***h(n)* = estimated cost from *n* to goal**
  - ***f(n)* = estimated total cost of path through *n* to goal**

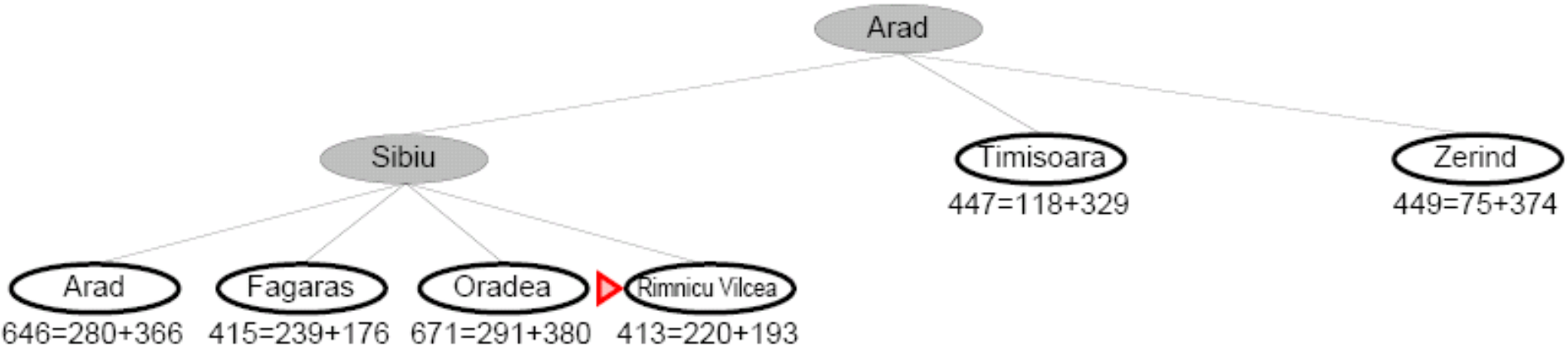
A* search example



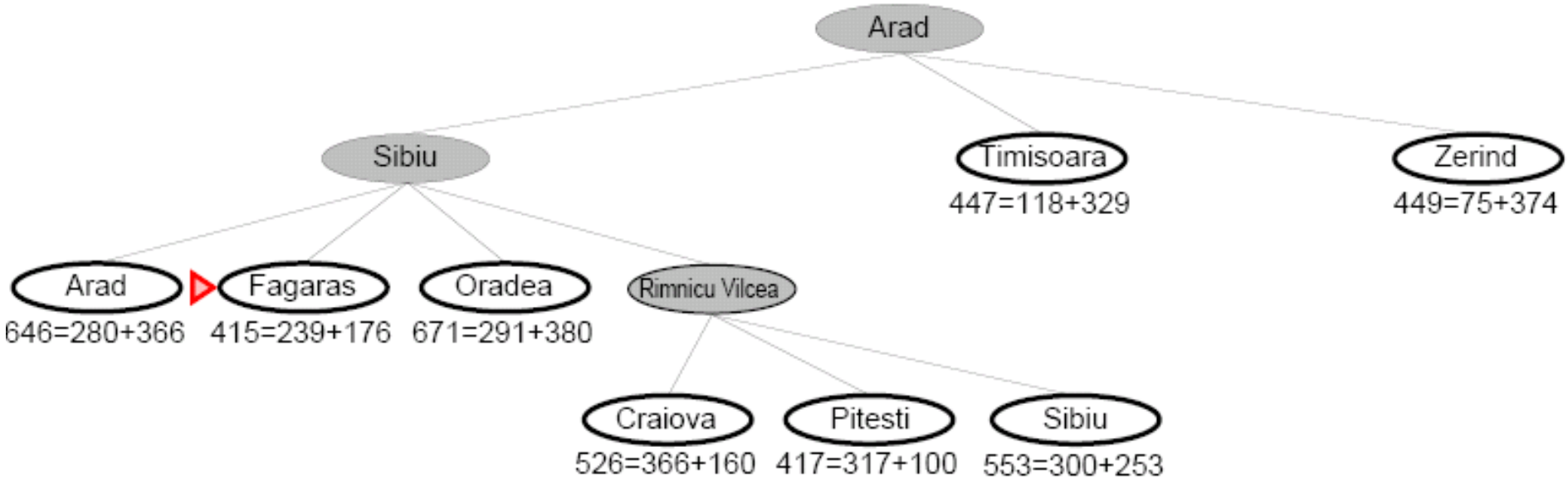
A* search example



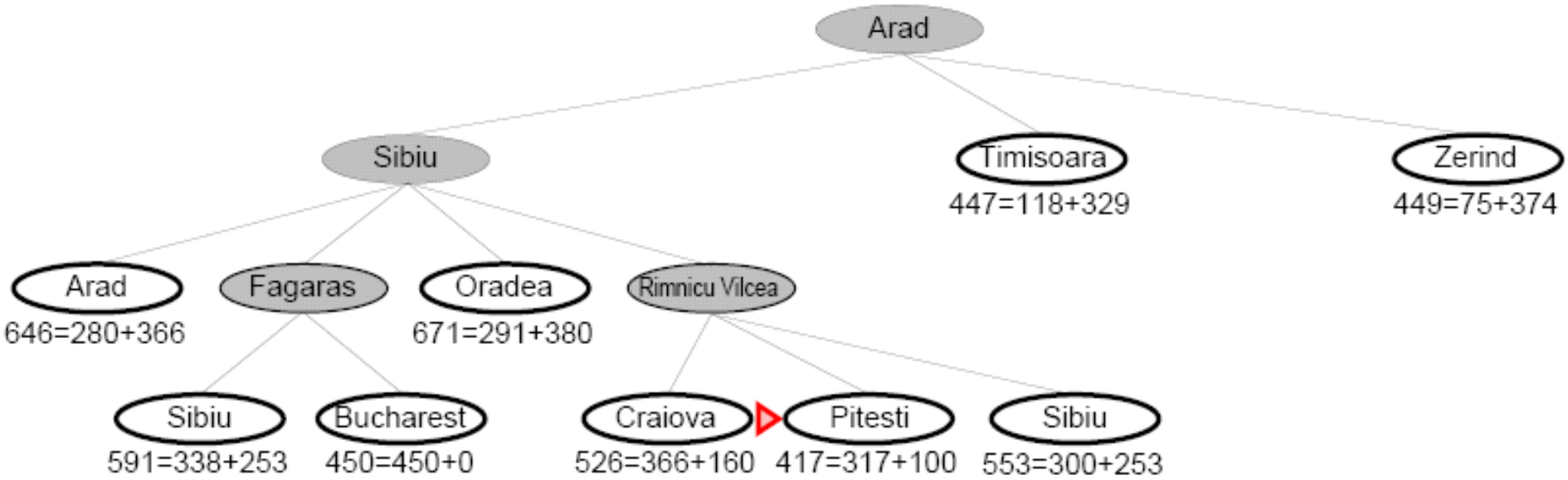
A* search example



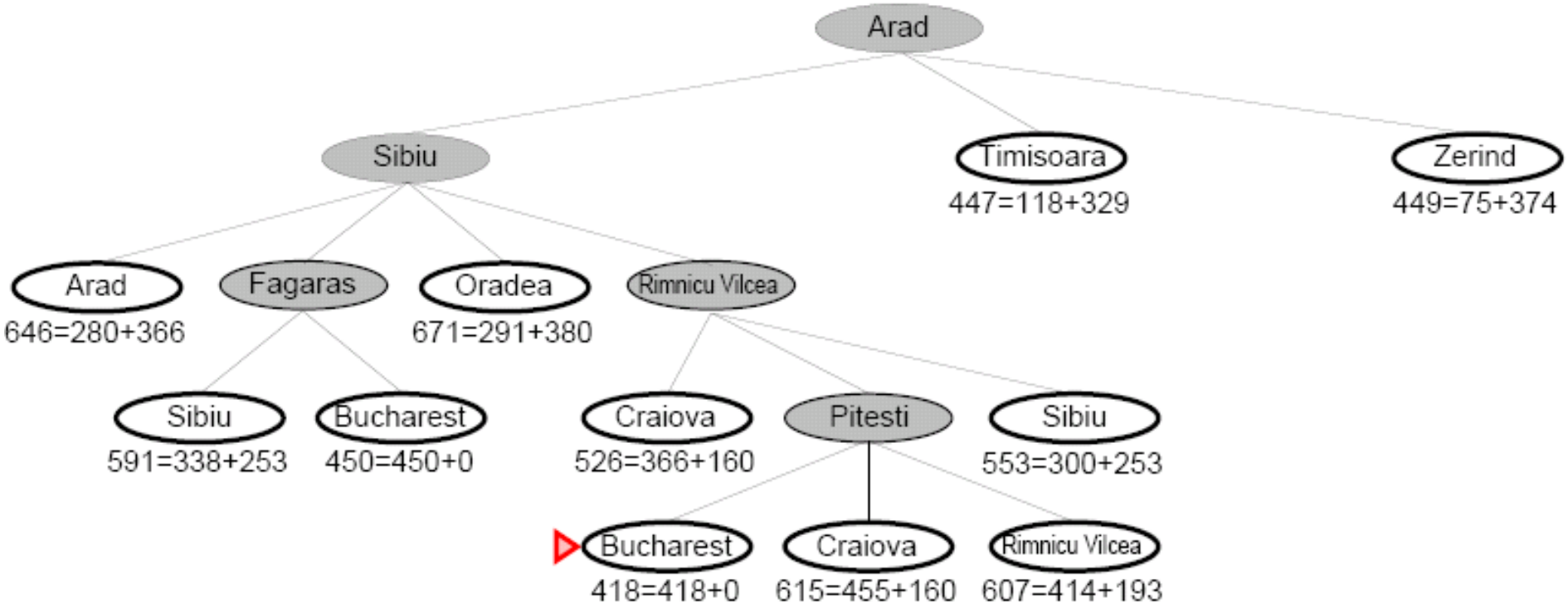
A* search example



A* search example



A* search example



Admissible heuristics

- A heuristic $h(n)$ is **admissible** if for every node n,
  $h(n) \leq h^*(n)$, where $h^*(n)$ is the **true** cost to reach the goal state from n.

- An admissible heuristic **never overestimates** the cost to reach the goal, i.e., it is **optimistic**

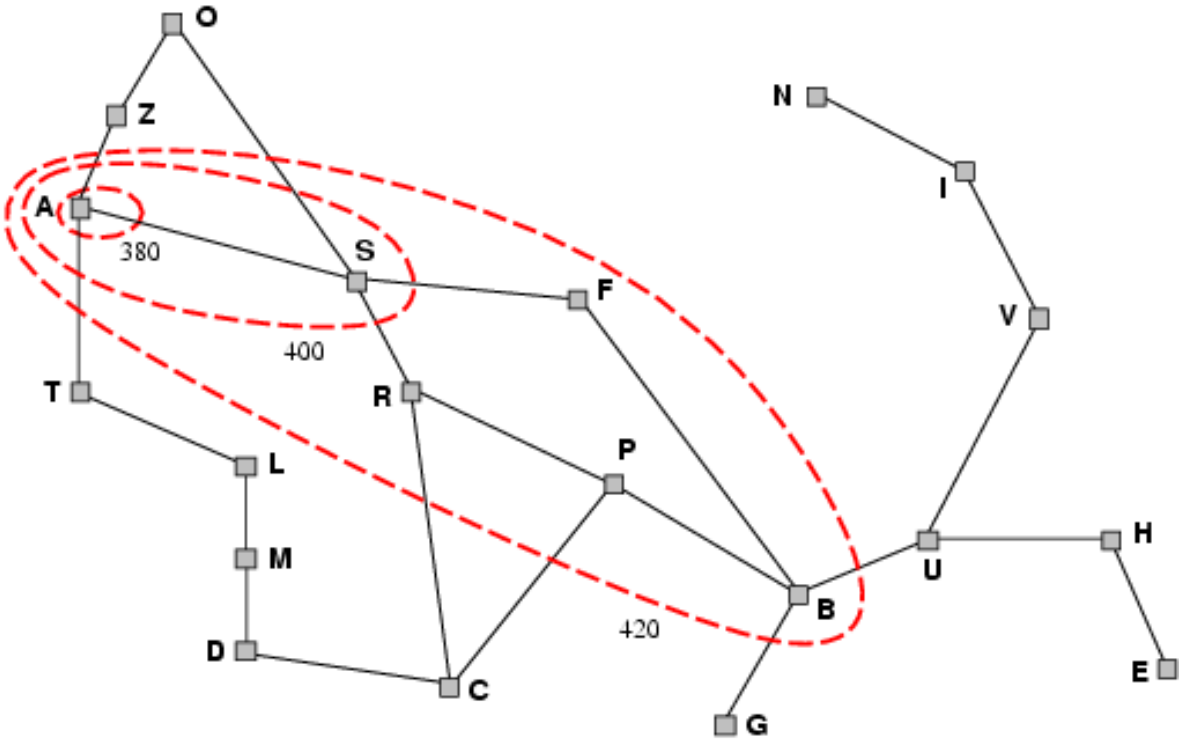
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

Optimality of A*

- **Theorem: If *h(n)* is admissible, A* guarantees to be optimal**

Optimality of A*

- **Lemma**: A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Tree search vs graph search

- There may exist many paths to the same state
- Tree search: No check for duplicate states
- Graph search: check for duplicate states in both closed and open lists
  - If a shorter path is found, need to reopen (if closed list) and update g cost and parent pointer.

Pseudo code of A* Algorithm

```
create the open list of nodes, initially containing only starting node
create the closed list of nodes, initially empty
while (we have not reached our goal) {
    consider the best node in the open list (the node with the lowest f value)
    if (this node is the goal) {
        then we're done;
    } else {
        move the current node to the closed list and consider all of its neighbors;
        for (each neighbor) {
            if (this neighbor is in the closed list and our current g value is lower) {
                move the node from closed list to open list;
                update the neighbor with the new, lower, g value;
                change the neighbor's parent to our current node;
            } else if (this neighbor is in the open list and our current g value is lower) {
                update the neighbor with the new, lower, g value;
                change the neighbor's parent to our current node;
            } else this neighbor is not in either the open or closed list {
                add the neighbor to the open list and set its g value;
            }
        }
    }
}
```

Properties of A*

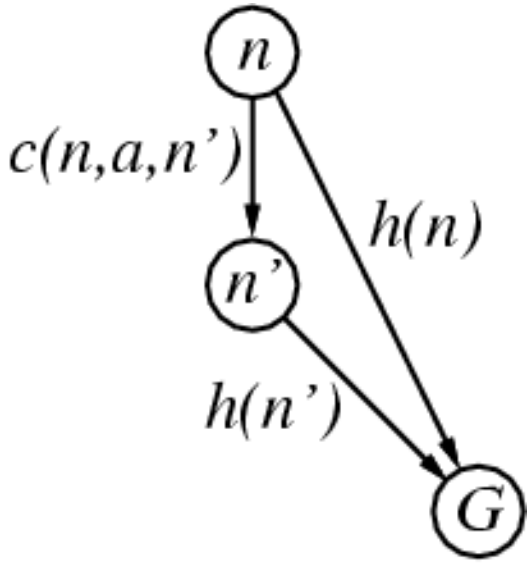
- Complete?
- Optimal?
- Time?
- Space?

Consistent heuristics

- **A heuristic is consistent (monotonic) if for every node *n*, every successor *n'* of *n* generated by any action *a*,**

  h(n) ≤ c(n,a,n') + h(n')

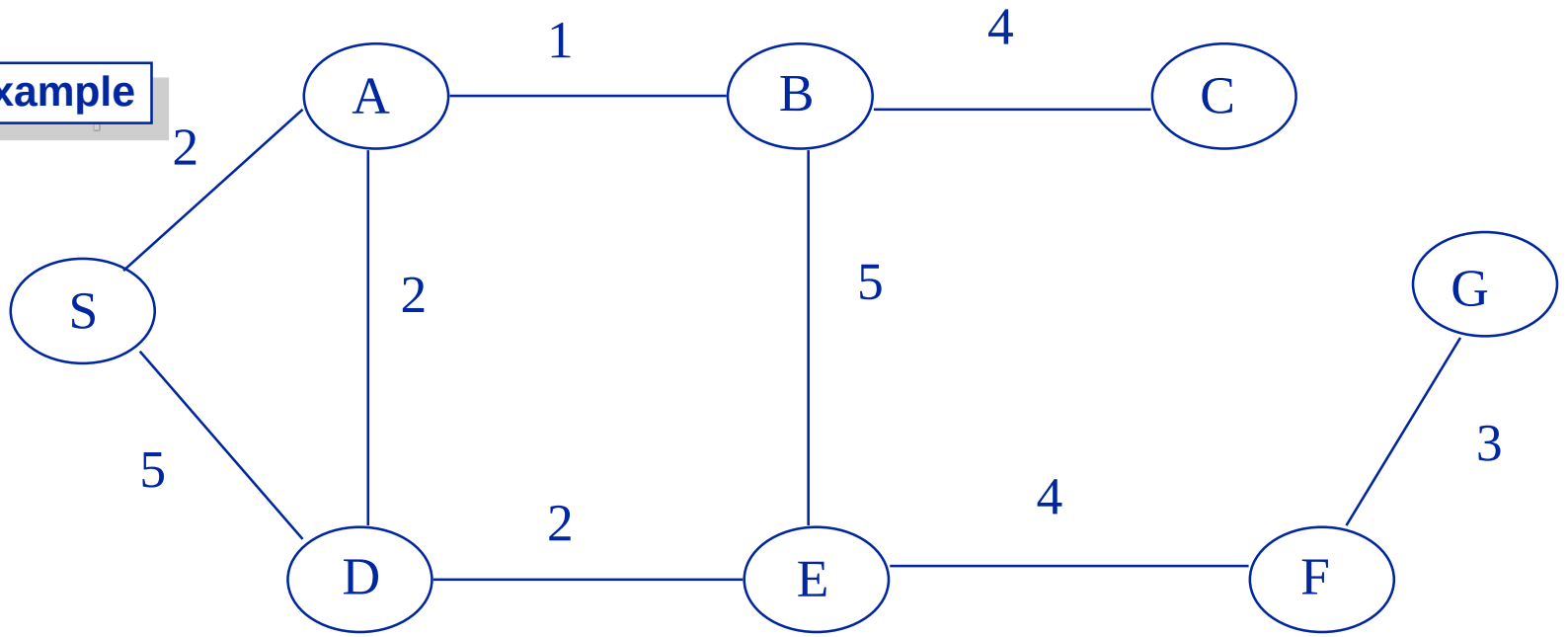
- **If *h* is consistent, *f(n)<=f(n')*.**

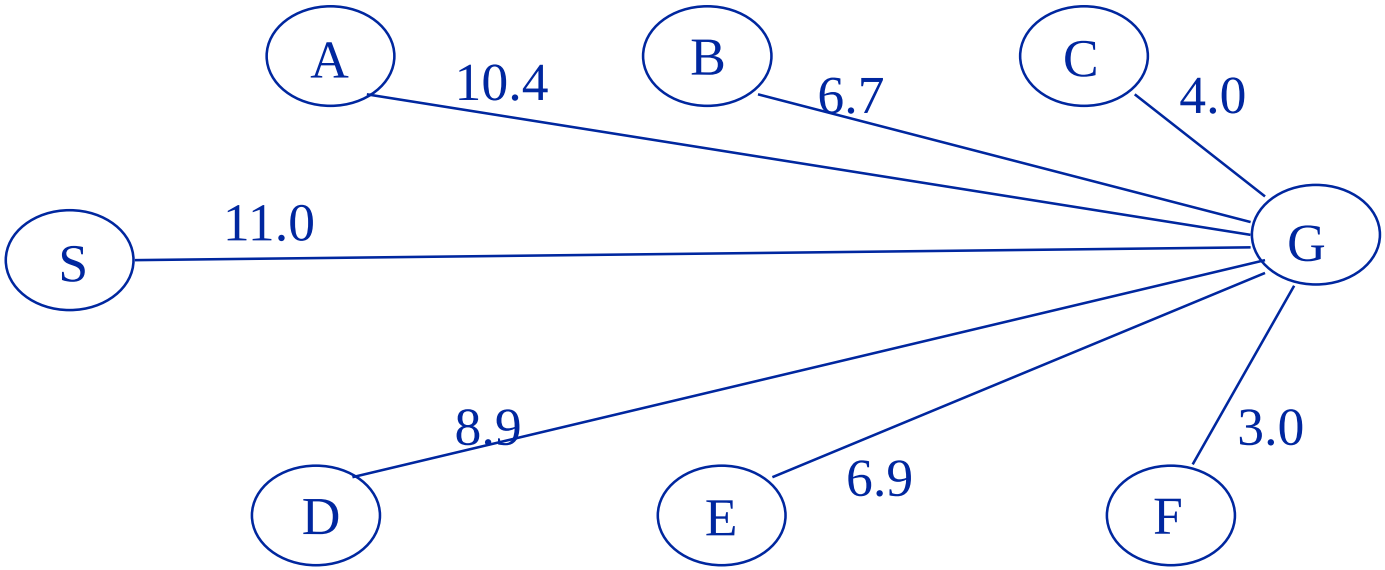


Optimality of A*

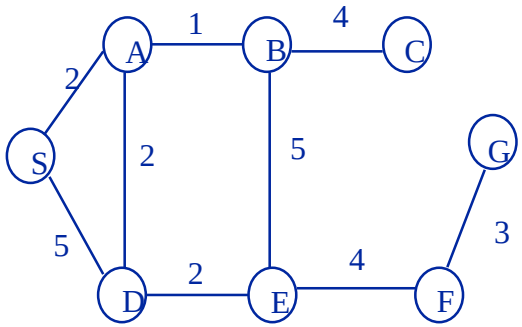
- **Theorem: If *h(n)* is consistent, A* guarantees to find an optimal path without reexpanding nodes**

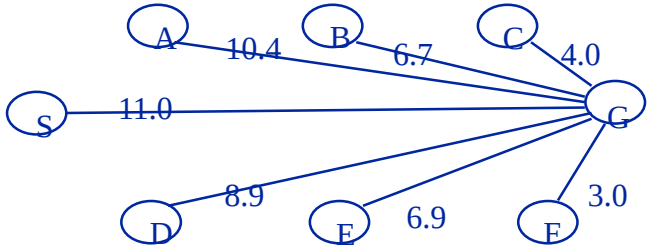
Example





A* Algorithm in action





Depth-first Branch and Bound Search

- Although A* guarantees to generate no more nodes than ***any*** other optimal algorithm, its space requirement can be prohibitive for large search problems
- Depth-first search algorithm can be enhanced to utilize heuristic
  - So called **Depth-first Branch and Bound Search**
  - Uses heuristic functions to bound solution quality, and only expand nodes that can still lead to solutions better than the incumbent

Depth-first Branch and Bound Search

```
Initialize:
Let open list Q = {S}
Let closed list C={}
L ← ∞ // score of best solution so far
While Q is not empty
        pull Q1, the first element in Q
        if Q1 is a goal compute the cost of the solution and update
           L <-- minimum between new cost and old cost
        else
                child_nodes = expand(Q1),
                For each child node n do:
                        Evaluate f(n).
                        If f(n) is greater than L , discard n.
                end-for
                Put remaining child_nodes on top of queue
        in the order of their evaluation function, f.
        end
Continue
```

Properties of Branch-and-Bound

- Complete?
- Optimal?
- Time?
- Space?

Iterative Deepening A* (IDA*)

- **Extend iteratively deepening search by**
  - **Using depth-first branch and bound**
  - **Updating the search limit according heuristic evaluations**

- **Properties:**

  Guarantee to find an optimal solution

  time: exponential, like A*

  space: linear, like B&B.

Iterative Deepening A* (IDA*)

- **Pseudocode:**

  Initialize: f <-- the evaluation function of the start node

  until goal node is found

  Loop:

  Do Branch-and-bound with upper-bound L equal current evaluation function

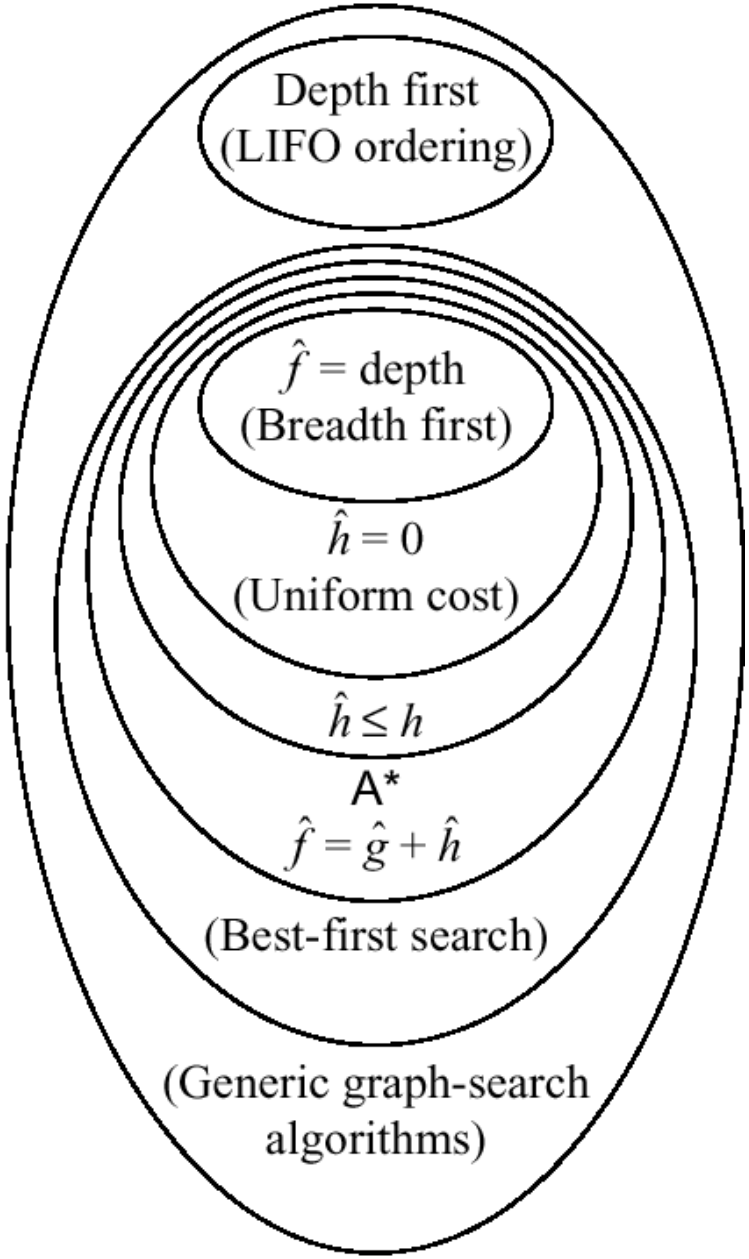
  Increment evaluation function to next contour level

  (Update evaluation function to the minimum f-value which exceeded f among states which were generated)

  end

  continue

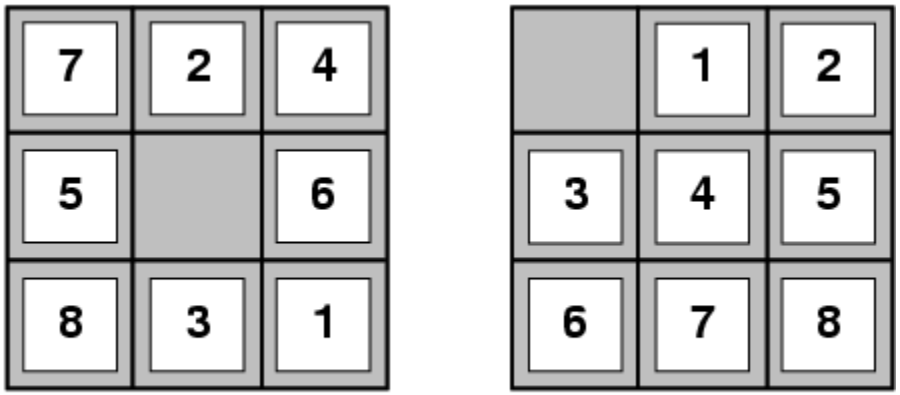
Relationships among search algorithms



Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ =
- $h_2(n)$ =



- $h_1(S)$ =
- $h_2(S)$ =

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
  then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Combination of Multiple Heuristics

- Given any admissible heuristics h_a, h_b,

  $h(n) = \max(h_a(n); h_b(n))$

  is also admissible and dominates h_a, h_b

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution