Artificial Intelligence

Introduction to Search

(Ch. 3.1-4)
Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
Problem types

- Deterministic, fully observable $\rightarrow$ single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence

- Non-observable $\rightarrow$ sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence

- Nondeterministic and/or partially observable $\rightarrow$ contingency problem
  - Percepts provide new information about current state
  - Solution is a contingent plan or a policy
  - Often interleave search, execution

- Unknown state and action space $\rightarrow$ exploration problem
Example: vacuum world

- **Single-state**, start in #5. Solution?

- **Sensorless**, start in \{1,2,3,4,5,6,7,8\} e.g., Right goes to \{2,4,6,8\}. Solution?
## Example: vacuum world

- **Contingency**
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: \([L, \text{Clean}]\), i.e., start in #5 or #7

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A problem is defined by four items:

1. initial state e.g., "at Arad"
2. actions or successor function $S(x) = \text{set of action–state pairs}$
   - e.g., $S(\text{Arad}) = \{<\text{Arad} \rightarrow \text{Zerind}, \text{Zerind}>, \ldots \}$
3. goal test, can be
   - explicit, e.g., $x = \text{"at Bucharest"}$
   - implicit, e.g., $\text{Checkmate}(x)$
4. path cost (additive)
   - e.g., sum of distances, number of actions executed, etc.
   - $c(x,a,y)$ is the step cost, assumed to be $\geq 0$

A solution is a sequence of actions leading from the initial state to a goal state.
Selecting a state space

- (Abstract) Real world is absurdly complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad" must get to some real state "in Zerind"
- (Abstract) solution = set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem
真空世界状态空间图

- states?
- actions?
- goal test?
- path cost?
Example: The 8-puzzle

- states?
- actions?
- goal test?
- path cost?

[Note: optimal solution of n-Puzzle family is NP-hard]
Example: robotic assembly

- states?
- actions?
- goal test?
- path cost?
Tree search algorithms

• Basic idea:
  – offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. expanding states)

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```
Tree search example
Tree search example
Tree search example
Implementation: general tree search

```plaintext
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERT-ALL(EXPEND(node, problem), fringe)

function EXPEND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node; ACTION[s] ← action; STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```
Implementation: states vs. nodes

• A **state** is a (representation of) a physical configuration

• A **node** is a data structure constituting part of a search tree includes state, parent node, action, path cost $g(x)$, depth

• The **Expand** function creates new nodes, filling in the various fields and using the **Successor-Fn** of the problem to create the corresponding states.
Search strategies

- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - optimality: does it always find a least-cost solution?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
- Time and space complexity are measured in terms of
  - $b$: maximum branching factor of the search tree
  - $d$: depth of the least-cost solution
  - $m$: maximum depth of the state space (may be $\infty$)
Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search
Breadth-first search

- Expand shallowest unexpanded node
- **Implementation:**
  - *fringe* is a FIFO queue, i.e., new successors go at end
Breadth-first search

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Properties of breadth-first search

- Complete?
- Optimal?
- Time?
- Space?
**Uniform-cost search**

- Expand least-cost unexpanded node
- **Implementation:**
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- **Complete?**
- **Optimal?**
- **Time?**
- **Space?**
Depth-first search

• Expand deepest unexpanded node

• Implementation:
  – fringe = LIFO queue, i.e., put successors at front
Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *fringe* = LIFO queue, i.e., put successors at front
Depth-first search

- Expand deepest unexpanded node
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Properties of depth-first search

- Complete?
- Optimal?
- Time?
- Space?
Depth-limited search

- Depth-first search with depth limit \( l \), i.e., nodes at depth \( l \) have no successors

- Recursive implementation:

```plaintext
function Depth-Limited-Search(problem, limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)

function Recursive-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if Goal-Test[problem](State[node]) then return Solution(node)
    else if Depth[node] = limit then return cutoff
    else for each successor in Expand(node, problem) do
        result ← Recursive-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```
Iterative deepening search

function ITTERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure

inputs: problem, a problem

for depth ← 0 to ∞ do
    result ← DEPTH-LIMITED-SEARCH(problem, depth)
    if result ≠ cutoff then return result
Iterative deepening search $l=0$
Iterative deepening search $l = 1$
Iterative deepening search $l = 2$
Iterative deepening search $l = 3$
Iterative deepening search

• Number of nodes generated in a depth-limited search to depth $d$ with branching factor $b$:
  \[ N_{DLS} = \]

• Number of nodes generated in an iterative deepening search to depth $d$ with branching factor $b$:
  \[ N_{IDS} = \]

• For $b = 10$, $d = 5$,
  \[ N_{DLS} = \]
  \[ N_{IDS} = \]

• Overhead =
Properties of iterative deepening search

- Complete?
- Optimal?
- Time?
- Space?
## Summary of algorithms

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Uniform-Cost</th>
<th>Depth-First</th>
<th>Depth-Limited</th>
<th>Iterative Deepening</th>
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<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time</td>
<td>$O(b^{d+1})$</td>
<td>$O(b^{C*/\epsilon})$</td>
<td>$O(b^m)$</td>
<td>$O(b^l)$</td>
<td>$O(b^d)$</td>
</tr>
<tr>
<td>Space</td>
<td>$O(b^{d+1})$</td>
<td>$O(b^{C*/\epsilon})$</td>
<td>$O(bm)$</td>
<td>$O(bl)$</td>
<td>$O(bd)$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
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Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!
Graph search

- The only difference is detecting repeated states

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem)(STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```
Summary

• Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

• Variety of uninformed search strategies

• Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

• Graph search can be exponentially more efficient than tree search