Lecture 2: Decision Trees

Machine Learning
Queens College
Today

• Decision Trees
  – Entropy and Information Theory
  – Information Gain
  – “Purity” or “Classification Accuracy”
Decision Trees

- Trees that define a decision process
  - Internal nodes: questions associated with a specific feature
  - Leaves: Decisions

```
Want a fast meal?
  Yes
  How about coffee?
    Yes
    Starbucks
    No
    McDonald’s
  No
  On expense account?
    Yes
    21 Club
    No
    T.G.I. Friday’s
```
Decision Trees

- Very easy to evaluate.
- Nested if statements
More formal Definition of a Decision Tree

- A **Tree** data structure
- Each **internal node** corresponds to a feature
- **Leaves** are associated with target values.
- Nodes with **nominal features** have $N$ children, where $N$ is the number of nominal values
- Nodes with **continuous features** have two children for values less than and greater than or equal to a **break point**.
Training a Decision Tree

• How do you decide what feature to use?
• For continuous features how do you decide what break point to use?

• Goal: Optimize Classification Accuracy.
# Example Data Set

<table>
<thead>
<tr>
<th>Height</th>
<th>Weight</th>
<th>Eye Color</th>
<th>Gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>170</td>
<td>Blue</td>
<td>Male</td>
</tr>
<tr>
<td>73</td>
<td>210</td>
<td>Brown</td>
<td>Male</td>
</tr>
<tr>
<td>72</td>
<td>165</td>
<td>Green</td>
<td>Male</td>
</tr>
<tr>
<td>70</td>
<td>180</td>
<td>Blue</td>
<td>Male</td>
</tr>
<tr>
<td>74</td>
<td>185</td>
<td>Brown</td>
<td>Male</td>
</tr>
<tr>
<td>68</td>
<td>155</td>
<td>Green</td>
<td>Male</td>
</tr>
<tr>
<td>65</td>
<td>150</td>
<td>Blue</td>
<td>Female</td>
</tr>
<tr>
<td>64</td>
<td>120</td>
<td>Brown</td>
<td>Female</td>
</tr>
<tr>
<td>63</td>
<td>125</td>
<td>Green</td>
<td>Female</td>
</tr>
<tr>
<td>67</td>
<td>140</td>
<td>Blue</td>
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Baseline Classification Accuracy

• Select the majority class.
  – Here 6/12 Male, 6/12 Female.
  – Baseline Accuracy: 50%

• How good is each branch?
  – The improvement to classification accuracy
Training Example

• Possible branches

```
<table>
<thead>
<tr>
<th>Color</th>
<th>M</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>brown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>green</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

```
M / F   M / F   M / F
```
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</table>
Training Example

• Possible branches

- Height < 68
  - 1M / 5F
  - 5M / 1F

  50% Accuracy before Branch
  83.3% Accuracy after Branch
  33.3% Accuracy Improvement
## Weight

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Training Example

• Possible branches

weight

<155

0M / 5F 6M / 1F

50% Accuracy before Branch
91.7% Accuracy after Branch
41.7% Accuracy Improvement
Training Example

- Recursively train child nodes.

```
weight
  <155
  5F
  >155
  6M / 1F

height
  <70
  2M / 1F
  >70
  4M
```
Training Example

• Finished Tree

- weight
  - <155
    - 5F
  - 6M / 1F

- height
  - <70
    - 2M / 1F
  - 4M

- color
  - blue
  - brown
  - green
Generalization

• What is the performance of the tree on the training data?
  – Is there any way we could get less than 100% accuracy?

• What performance can we expect on unseen data?
Evaluation

• Evaluate performance on data that was not used in training.
• Isolate a subset of data points to be used for evaluation.
• Evaluate generalization performance.
Evaluation of our Decision Tree

• What is the Training performance?
• What is the Evaluation performance?
  – Never classify female over 165
  – Never classify male under 165, and under 68.
  – The middle section is trickier.
• What are some ways to make these similar?
There are many pruning techniques. A simple approach is to have a minimum membership size in each node.
Decision Trees

- Training via **Recursive Partitioning**.
- Simple, interpretable models.
- Different node selection criteria can be used.
  - Information theory is a common choice.
- Pruning techniques can be used to make the model more robust to unseen data.
Entropy and Information Theory

• Entropy is a measure of how homogenous a data set is.
  – Also how even a probability distribution or a random variable is.
• The unit of Entropy is the **bit**.
• Under an Information Theory perspective entropy represents the fewest bits it would take on average to transmit information in a signal (i.e. a random variable)
Entropy

• Say I have a vocabulary of 4 items.
  – A, B, C, D.
• A standard encoding of these might be
  – 00, 01, 10, 11.
• 2 bits per vocabulary item.
• However, if A is much more common, it might be more efficient to use this coding
  – 0, 10, 111, 110
• Exercise: What is the average bit length if there are 150 As, 40 Bs, 5 Cs, and 5Ds?
Calculating Entropy

\[ H(X) = - \sum_{i \in X} p_i \log p_i \]

- Where \( p_i \) is the probability of selecting the \( i \)th value.
- For example, say \( X = \{A A A B B B B B B\} \)
- In the calculation of entropy \( 0 \log 0 = 0 \)

\[ H(X) = - \left( \frac{3}{8} \log \frac{3}{8} + \frac{5}{8} \log \frac{5}{8} \right) \]
Information Gain

- In our previous example we examined the improvement to classification performance.
  - Error reduction or change to overall accuracy.
- Using entropy the measure that is optimized is Information Gain.
  - The difference in the entropy of the label or class distribution before or after a particular decision tree split.
Calculating Information Gain

\[
Gain(S, F) = H(S) - \sum_{f \in values(F)} \frac{|S_f|}{|S|} H(S_f)
\]

\[
H(S) =
\]

\[
H(S_{VW}) =
\]

\[
H(S_{Ford}) =
\]

\[
H(S_{BMW}) =
\]

\[
Gain(S, F) =
\]
Calculating Information Gain

$$Gain(S, F) = H(S) - \sum_{f \in values(F)} \frac{|S_f|}{|S|} H(S_f)$$

Identify the feature with the greatest Information Gain and repeat this process recursively!
Visualization of Decision Tree Training
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Next Time: Math Primer

• Probability
  – Bayes Rule
  – Naïve Bayes Classification

• Statistics
  – Normal Distribution
  – Multinomial Distribution